THE
TWO DOZEN
RIGHT TRIANGLES
OF A
SQUARE PYRAMID

NEAL BERNTSON
Empire State College
<table>
<thead>
<tr>
<th>DAY</th>
<th>TOPIC</th>
<th>OBJECTIVE</th>
<th>CLASSROOM ITEMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Division of a Square into 8 Right Triangles</td>
<td>Manipulating a square base of a pyramid yields several triangles if a half-division technique is performed.</td>
<td>Card Stock, Ruler, Scissors, Color Markers</td>
</tr>
<tr>
<td></td>
<td>Hypotenuse x 2 = Diagonal</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>&quot;Slant Height&quot;</td>
<td>Altitudes</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Division of a Pyramid's 4 sides into 8 Right Triangles</td>
<td>Comprehend bisecting the sides of any pyramid will produce two times the number of triangles and all of them will be right.</td>
<td>Scotch Tape, Card Stock, Ruler, Scissors, Color Markers</td>
</tr>
<tr>
<td></td>
<td>&quot;Slant Height&quot;</td>
<td>&quot;Altitudes&quot;</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Design of a Vertical Right Triangle inside Pyramid fitting into the Corner.</td>
<td>Understand how a see-through pyramid will demonstrate how a new interior right triangle is to be called the &quot;CORNER - SUPPORT&quot;</td>
<td>Transparencies, Card cut in Right Triangle, Scotch Tape, Ruler, Scissors</td>
</tr>
<tr>
<td></td>
<td>&quot;Lateral Edge&quot;</td>
<td>Hypotenuse</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Design of a Vertical Right Triangle inside Pyramid fitting onto the Side.</td>
<td>Understand how a see-through pyramid will demonstrate how a new interior right triangle is to be called the &quot;SIDE - SUPPORT&quot;</td>
<td>Transparencies, Card cut in Right Triangle, Scotch Tape, Ruler, Scissors</td>
</tr>
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<td></td>
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<td>Hypotenuse</td>
<td></td>
</tr>
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<td>DAY</td>
<td>TOPIC</td>
<td>OBJECTIVE</td>
<td>CLASSROOM ITEMS</td>
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<td>-----</td>
<td>--------------------------------------</td>
<td>---------------------------------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>5</td>
<td>Finding the Mystery Segment</td>
<td>Demonstrate most efficient use of the Pythagorean Theorem to find length of any of the pyramid line segments.</td>
<td>Calculator</td>
</tr>
<tr>
<td></td>
<td>* * * * *</td>
<td></td>
<td>Pyramid</td>
</tr>
<tr>
<td></td>
<td>Most Direct Strategy Identification</td>
<td></td>
<td>Constructions</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ruler for</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Estimations</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>White-Board</td>
</tr>
</tbody>
</table>

**First Lesson: "DIVIDING A SQUARE"**
Can a student see the many options from a folded-up square to impose the Pythagorean Theorem in several configurations? Will he or she observe the symmetry of this pattern?

**Second Lesson: "TRIANGULAR FACES"**
Can a student repeat the division of each lateral face on a constructed pyramid and apply the properties of this isosceles triangle to expand on what dimensions the pyramid holds?

**Third Lesson: "VERTICAL TRIANGLE: CORNER - SUPPORT"**
Can a student begin to identify and tally the many right triangles available at their disposal by visualizing the interior options of a pyramid’s many characteristics?

**Fourth Lesson: "VERTICAL TRIANGLE: SIDE - SUPPORT"**
Can a student continue to construct one last option of a vertical right triangle on the inside of a pyramid that will be equally as strategic in discovering line segment lengths?

**Fifth Lesson: "PYTHAGOREAN PRODUCTIVITY"**
Can a student perform the most strategic and productive process to solve a line segment length armed with the tools of algebra, terminology, visual aids, and spatial orientation?
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>BELL RINGER</td>
<td>Identify the legs and hypotenuse of right triangles. (3-4-5, 5-12-13, 8-15-17, 7-24-25, 9-40-41) Filling in the one missing number to match the other 2 given.</td>
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<tr>
<td>STRETCH - OUT</td>
<td>How many ways are there to split a square in half with one line? Demonstrate these divisions by folding card stock into a square and then subdividing with creases. Darken lines with markers.</td>
</tr>
<tr>
<td>KICK - OFF</td>
<td>Students color in the newly formed right triangles. Explore how measurements of the square's characteristics coincide with right triangles' segments.</td>
</tr>
<tr>
<td>REVIEW</td>
<td>Half the square's diagonal is a hypotenuse of any of the right triangles and half the square's side is the shorter leg to any of the right triangles.</td>
</tr>
<tr>
<td>OVERTIME</td>
<td>How many right triangles total? How many are identical? How many are mirror images to one another? If you rotate around the square's perimeter clock-wise, what is the pattern of right triangles versus mirror images?</td>
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<tr>
<td>HOMEWORK</td>
<td>Find examples of pyramids in your community and describe the differences between them (Structural or Decorative? Flat or Tall? Square Base or Rectangular Base? See-through or Solid?) Also, make ten sketches of a pyramid free-hand. No Tracing! Show the outline to be hollow so that all edges can be seen.</td>
</tr>
</tbody>
</table>
Bell Ringer: Question: “What differences exist between the triangle’s side lengths?”
Question: “How would you write a computer program to calculate these?”
Transition: “Think about how to create a hypotenuse out of a big square.”

Stretch-Out: Question: “How can you create a square from folding a rectangular card?”
Question: “How many ways can a card fold perfectly in half?”
Question: “How would you illustrate the creases before folding the card?”
Transition: “Share your folded card with a partner to observe the creases.”

Kick-Off: Question: “What would cause a different amount of right triangles?”
Question: “How is it possible some shapes are half the size of another?”
Transition: “Think how many times you could label any hypotenuse.”

Review: Question: “How would you describe the orientation of all the triangles?”
Question: “Why do mirror images show half of the square’s side length?”
Transition: “Think if there is any use for the two largest right triangles.”

Overtime: Question: “How might coloring triangles help understanding orientation?”
Question: “Does it matter where you begin rotating around the triangles?”
Transition: “Think about the convenient symmetry these 8 triangles have.”
2nd LESSON:

Bell Ringer: Question: “How can a rectangular pyramid’s base be drawn differently?”
Question: “How many ways can you draw a regular square pyramid?”
Transition: “Imagine how the base turns into a pyramid.”

Stretch-Out: Question: “What is another word for pyramid?”
Question: “Who can draw a triangular tetrahedron?”
Transition: “Think how there is no right-side-up for this pyramid.”

Kick-Off: Question: “How can we make a pyramid that has a definite top point?”
Question: “Why do you need two rulers to draw an isosceles triangle?”
Transition: “Think about the right triangle’s hypotenuse available to you.”

Review: Question: “How will the altitude of the triangle be a part of the pyramid?”
Question: “Do all the altitudes meet at the top point of the pyramid?”
Transition: “Think about the equation for the area of a triangle.”

Overtime: Question: “What is the equation for area for two identical right triangles?”
Question: “How many isosceles triangles make up the pyramid’s surface?”
Transition: “Try to calculate the total surface area without a calculator.”
3rd LESSON:

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<tr>
<td>BELL RINGER</td>
<td>How long is the lateral edge if the base of the square is 6 and the lateral face triangle has a height of 4. Also, if the base is 10 and the triangle has a height of 12 units?</td>
</tr>
<tr>
<td>STRETCH - OUT</td>
<td>How many right triangles are created in a regular square pyramid so far? Include all divisions on the base square surface and the bisecting of the four lateral face triangles.</td>
</tr>
<tr>
<td>KICK - OFF</td>
<td>Create a square base layout where the diagonal is 6&quot; in length. Cut out a (3 x 4 x 5) special triangle card to be taped vertically. Transparency sheets will be tricky to cut. With two rulers, cut four isosceles triangles with a base identical in length to the side of the square base and the legs are 5” each. Tape together two consecutive sides with the vertical triangle supporting between them. Tape the other 3 edges completing the other outside half.</td>
</tr>
<tr>
<td>REVIEW</td>
<td>What is the relationship between the internal vertical &quot;corner-support&quot; triangle's base and the square base's diagonal measure? How many more right triangles do we have?</td>
</tr>
<tr>
<td>OVERTIME</td>
<td>List any line segments that have two different names. Which ones are half or double the lengths of the others?</td>
</tr>
<tr>
<td>HOMEWORK</td>
<td>Given the new clear see-through pyramid constructed: How long is the base of the vertical corner-support right triangle if the area of the square base of the pyramid is 100 sq. inches?</td>
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</tbody>
</table>
Bell Ringer: Question: “How many different triangles can this edge be a hypotenuse?”

Transition: “Think of the lateral side as two adjacent equal right triangles.”

Stretch-Out: Question: “How many slanted right triangles are there?”

Question: “How many horizontal right triangles are there?”

Question: “How many total right triangles do we have so far?”

Transition: “Think how one of the lateral edges could be the hypotenuse to a vertical right triangle somewhere else in the pyramid.”

Kick-Off: Question: “How many vertical right triangles are there?”

Question: “Will the (3 x 4 x 5) triangle, fit slanted, change the pyramid?”

Transition: “Think how the pyramid's size would have to change.”

Review: Question: “What is the base length also the measure of?”

Question: “Is there only one place for this corner support to fit inside?”

Transition: “Find the total number of right triangles we have designed.”

Overtime: Question: “What’s splitting the square base’s diagonal create in length?”

Question: “Which line segments have more than one name that we use?”

Transition: “Think of the square base as two large right triangles again.

The Pythagorean Theorem can give us the diagonal’s length.”
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<td><strong>BELL RINGER</strong></td>
<td>Review homework problem's two possible approaches for the choice of which right triangle to investigate. The large has legs 10&quot; long. The small has legs 5&quot; long. Finding the diagonal of the large requires it be split in half to be the triangle's base length.</td>
</tr>
<tr>
<td><strong>STRETCH - OUT</strong></td>
<td>Where might the last four right triangles be configured in or on the square pyramid. With eight on bottom, eight on the sides, and four inside, is there any more room for more right triangles?</td>
</tr>
<tr>
<td><strong>KICK - OFF</strong></td>
<td>Create a square base layout where the sides are 6&quot; in length. Cut out a (3 x 4 x 5) special triangle card to be taped vertically. Transparency sheets will be cut with only one ruler this time. Cut four isosceles triangles with a 6&quot; base. At the mid-point, measure upward for an altitude of 5&quot; each. Tape together one side with the vertical triangle pressing a line that supports and divides the triangle. Tape the other 4 edges.</td>
</tr>
<tr>
<td><strong>REVIEW</strong></td>
<td>What is the relationship between the internal vertical &quot;side-support&quot; triangle's base and the square's side?</td>
</tr>
<tr>
<td><strong>OVERTIME</strong></td>
<td>With a hollow square pyramid image, identify two names for every line segment. Answers may include the factors of &quot;is half of&quot; or &quot;is twice the&quot; for all line segments interrelations.</td>
</tr>
<tr>
<td><strong>HOMEWORK</strong></td>
<td>Write two &quot;Find the Length of a Line Segment&quot; story problems using proper vocabulary in identifying the names of all the lines inside and out of a regular square pyramid. Sketch a pyramid and give one measurement and ask for another where the answer will be twice as much. The other question must give one measurement where the answer will be half as much in length.</td>
</tr>
</tbody>
</table>
Bell Ringer:  Question: “Are there two methods to solving the homework problem?”
Question: “Is one easier than the other?”
Transition: “Think of the only remaining spot to design the last triangles.”

Stretch-Out:  Question: “Can we still position right triangles upon the exterior?”
Question: “Is there anywhere left on the interior to construct a triangle?”
Transition: “Think of the pyramid’s slant height as a seam for an edge.”

Kick-Off:  Question: “Do you need two rulers to construct the isosceles triangle?”
Question: “What will the slant height be to match the hypotenuse length?”
Transition: “Compare the triangle base’s usefulness with other examples.”

Review:  Question: “Can you calculate the pyramid’s square base area only knowing the base length of one of the vertical triangles?”
Question: “What else would you need to calculate the lateral area?”
Transition: “Identify line segments with pairs of names.”

Overtime:  Question: “Who can mark every line segment term?”
Transition: “Think how a length leads to another triangle’s line segment.”
### 5th LESSON:

<table>
<thead>
<tr>
<th>TASK</th>
<th>DETAILS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BELL RINGER</strong></td>
<td>Review all vocabulary: Base Edge, Diagonal, Lateral Edge, Slant Height, Altitude, Vertical &quot;Corner - Support&quot; Right Triangle, Vertical &quot;Side - Support&quot; Right Triangle, Hypotenuse</td>
</tr>
<tr>
<td><strong>STRETCH - OUT</strong></td>
<td>Review all constructed pyramids and relevant vocabulary for each. Depending on featured elements, all terms do not apply.</td>
</tr>
<tr>
<td><strong>KICK - OFF</strong></td>
<td><strong>PARTNERS - QUIZ</strong>&lt;br&gt;Assessment includes students answering each other's partner’s homework's story problems to be written on index cards. When the pairs complete the quiz and participate in one of the two games, the collected index cards are shuffled and recited for the Round-Robin which reinforces student-participation of data.</td>
</tr>
<tr>
<td><strong>REVIEW</strong></td>
<td><strong>ROUND - ROBIN:</strong> With a projected image of a hollow regular square pyramid, students draw a right triangle contained in the figure. Passing a different colored dry-erase marker to another, the next classmate must draw a new right triangle not of the same genre. This triangle must share a line with the previous.</td>
</tr>
<tr>
<td><strong>OVERTIME ACTIVITY</strong></td>
<td><strong>HUMAN – PYRAMID:</strong> With four strings from a point in the ceiling reaching to mark a square on the floor, four students sit inside each in their own quadrant triangle. When a line segment name is given and another name is to be found, students must each hold up a mini-triangle to show which choice of right triangle they project upon that would solve the line segment length. If the answer is “corner-support” and students gesture toward corners that squeeze a slower responding student out of space, then they are out of the game and another student sits in.</td>
</tr>
</tbody>
</table>
Bell Ringer:  Question: “How many total right triangles are on and inside the pyramid?”
Question: “Can you categorize them all and put them into groups?”
Transition: “Sit with your partner and get out your homework.”

Stretch-Out:  Question: “Do you have all pyramid models handy and an index card?”
Question: “Do you understand your partner’s homework story problem?”
Transition: “Try not to spend more than three minutes on any problem.”

Kick-Off:  Question: “Do you have your partner’s homework story problem written on an index card to be handed-in for use in the Round-Robin?”
Transition: “When you’re done, you can play Human - Pyramid.”

Review:  Question:  (example) “If I know the square base’s diagonal and the altitude, how can I find the length of a lateral edge?”
Question:  (example) “If I know the square’s base’s side length and the slant height, how can I find the pyramid’s altitude?”
Transition: “Display your best hand-made pyramid model.”
Square Pyramid

The base is a square.

A square pyramid is a pyramid with a square base.
Square Pyramid

**TOTAL SURFACE AREA OF A PYRAMID**

1. **Step One**
2. **Step Two**
3. **Step Three**
The letter "s" is the length of a side of the square base.

The letter "h" is the altitude of the pyramid.

The letter "l" is the slant height. There is a reason this height is called slant height!

The word slant refers to something that is oblique or bent.

Something that is not vertical or straight up.

Basically, anything that is not horizontal or vertical!

The letter "L" can also be thought of as the altitude of the slanted triangle.

It starts from center of side "s" and goes straight up…

to the peak of the isosceles triangle.
The length of the Lateral Edge can be found by using the Pythagorean Theorem.

Half of "s" is 5 inches which is the base leg. 12 inches is the other leg.

The square of the Lateral Edge's length is equal to the sum of 25 and 144.

Half of "s" is 5.5 inches which is the base leg. 13 inches is the other leg.

The square of the Lateral Edge's length = (5.5)^2 + 169.
Triangle ABC is the vertical "Corner - Support" right triangle.

Line "a" is the pyramid's altitude or height.

Line "b" is the hypotenuse of the triangle and the Lateral Edge of the pyramid.

Line BC is half the square base's diagonal.

The red triangle is the vertical "Side - Support" right triangle.

The hypotenuse "l" is also the slant height.
1.) Vocabulary: What terminology has been used during class discussion to describe the positioning of the shaded triangle pictured above? _________________________

2.) What is the area of the base in square centimeters? ____________

3.) What is the length of line segment "Ap" pictured above? ____________

4.) What is the length of the diagonal of the base square? ____________

************************************************************************

5.) Extension: Select and describe how two of the sides in the shaded right triangle are each part of any other type of positioned right triangle in the square pyramid.

SIDE ________ IS ALSO ________________________________________________

_______________________________________________________________________

SIDE ________ IS ALSO ________________________________________________

_______________________________________________________________________
h = Height or Altitude
s = Slant Height (usually called letter "L")
a = Side Length (usually called letter "s")
e = Lateral Edge Length

6.) Given: Slant Height = 20 and Altitude = 16. Find: "a" ____________

7.) Given: Slant Height = 10 and "e" = 12. Find: "a" ____________

8.) Given: Side Length = 5 and "e" = 8. Find: Slant Height ____________

9.) Given: Altitude = 15 and "e" = 17. Find: Slant Height ____________

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10.) Construction: In the space provided to the right of the above diagram, sketch the outline of an identical pyramid using lines “a” and “e” and “h” as labels again. DRAW another type of right triangle positioned on the interior of the pyramid.